DETERMINATION OF THE NUMBER OF SIDES OF A REGULAR POLYGON

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— SUMMARY —

IT'S VERY IMPORTANT FOR ALL OF US TO KNOW THE NUMBER OF SIDES OF A REGULAR POLYGON IN APPLIED GEOMETRY. THE BOOKS ADOPTED BY 1ST DEGREE SCHOOLS USUALLY PRESENTS A SIMPLE AND DIRECT FORMULA TO ITS DETERMINATION, BASED ON THE CENTRAL ANGLE. HOWEVER, WHEN WE HAVE SOME GEOMETRICAL PROBLEMS WITH UNKNOWN CENTRAL ANGLE, BUT KNOWING THE RATIO "r" BETWEEN THIS ON AND THE ADJACENT INNER ANGLE, WE'LL FIND A SOLUTION BY USING THE THEOREM, AS ANNOUNCED BELOW, WHOSE DEMONSTRATION IS THE MAIN POINT OF THIS WORK:

"IN A REGULAR POLYGON, THE NUMBER OF SIDES "n" IS THE DOUBLE OF THE RATIO "r" BETWEEN THE INNER ANGLE AND THE CENTRAL ANGLE, PLUS 2 (TWO), AS BELOW:

$$n = 2r + 2$$

WHEN THE SOLUTION REQUIRES THE DETERMINATION OF THE RATIO "r" BETWEEN THE INNER ANGLE AND THE CENTRAL ANGLE, THE FOLLOWING PROPERTY MUST BE USED:

"THE RATIO "r" BETWEEN THE INNER ANGLE AND THE CENTRAL ANGLE OF A REGULAR POLYGON IS THE HALF OF THE NUMBER OF SIDES "n", MINUS 1 (ONE), AS BELOW:

$$r = \frac{n}{2} - 1$$

1 - INTRODUCTION

THE DEMONSTRATION OF THE THEOREM REFERING TO THIS WORK WAS ORIGINATED DURING A MATHEMATIC CLASS APPLIED TO GROUP D4, FORM 8, 1ST DEGREE STUDENTS IN PORTO ALEGRE MILITARY SCHOOL, RIO GRANDE DO SUL IN THE BEGINING OF OCTOBER, 1985, OCCASION IN WHICH SEVERAL GEOMETRICAL PROBLEMS FROM THE TEXT BOOK IN REFERENCE WERE BEING INDIVIDUALLY RESOLVED BY THE STUDENTS.

ON THAT OPPORTUNITY, I DISCUSSED WITH THE TEACHER ABOUT SOME CONCLUSIONS DEDUCTED DURING THE SOLUTION OF A SAID PROBLEM, WHEN I WAS ORIENTED TO A RESEARCH OF THE CONCLUSIONS SUPPOSINGLY APPLICABLE, AND PRESENT THE RESULT IN A WRITTEN FORM.

THE HAND-WRITTEN ORIGINAL WORK WAS DELIVERED TO THE MATHEMATIC TEACHER, PROFESSOR FURLAN WHO, AFTER FERIFYING THE WORK, SENT IT TO THE HEADMASTER, LT. COLONEL ALEXANDRE MASCARELLO, WHO, ACCORDING TO HIM, HE DID EXHAUSTING RESEARCHES IN LIBRARIES AND SPOKED TO MANY TEACHERS, INTENDING TO COLLECT DATA TO ATTEST THE AUTHENTICITY OF THIS WORK.

THE ACKNOWLEDGEMENT OF IT WAS FORMALIZED THROUGH A NER 7 DOCUMENT OF THE TEACHING DIVISION, ON NOV 25, 1985 (ANNEX), PUBLISHED IN BULLETIN FROM THE MILITARY COLLEGE OF PORTO ALEGRE, RIO GRANDE DO SUL.

2 - DEMONSTRATION

2.1 - THEOREM OF THE NUMBER OF SIDES OF A REGULAR POLYGON

2.1.1 - STATEMENT

"IN A REGULAR POLYGON, THE NUMBER OF SIDES IS EQUAL TO THE DOUBLE OF THE RATIO BETWEEN THE INNER ANGLE AND THE CENTRAL ANGLE, PLUS 2 (TWO)."

2.1.2 - HYPOTHESIS

IT'S KNOWN THAT, IN A REGULAR POLYGON, THE SUM BETWEEN THE INNER ANGLE AND THE CENTRAL ANGLE IS EQUAL TO 180 DEGREES.

[illegible] $= 180^\circ$ (1)

IT'S ALSO KNOWN THAT THE CENTRAL ANGLE IS EQUAL TO:

$$\hat{CA} = \frac{360}{n} \qquad (2)$$

WHERE "n" IS THE NUMBER OF SIDES OF THE POLYGON.

2.1.3- THESIS

BEING "r" THE RATIO BETWEEN THE INNER ANGLE AND THE CENTRAL ANGLE OF A REGULAR POLYGON.

$$r = \frac{\hat{IA}}{\hat{CA}} \qquad (3)$$

TAKE THE VALUE OF $\hat{IA}$ IN THE EQUATION ABOVE (3) AND SUBSTITUTE IN THE EQUATION (1). SO, WE HAVE:

$$\hat{IA} = r \cdot \hat{CA}$$

$$r \cdot \hat{CA} + \hat{CA} = 180^\circ$$

$$\hat{CA}\,(r + 1) = 180^\circ$$

$$\hat{CA} = \frac{180^\circ}{r+1} \qquad (4)$$

EQUALIZING BOTH EQUATIONS (2) AND (4), WE GET:

$$\frac{180^\circ}{r+1} = \frac{360^\circ}{n}$$

$$180^\circ \cdot n = 360^\circ\,(r+1)$$

$$n = \frac{360^\circ}{180^\circ} \cdot (r+1)$$

$$n = 2 \cdot (r+1)$$

$$n = 2r + 2 \qquad (5)$$

FROM EQUATION (5), WE GET AN EXPRESSION TO DEFINE THE RATIO "r" BETWEEN THE INNER ANGLE AND THE CENTRAL ANGLE, AS BELOW:

$$r = \frac{n}{2} - 1 \qquad (6)$$

THROUGH THE EXPRESSION ABOVE (6), WE ELABORATED THE TABLE NUMBER 1 (ONE), ACORDING TO SOME EXAMPLES BELOW:

a) TRIANGLE

$$r = \frac{n}{2} - 1 \qquad r = \frac{3}{2} - 1 \qquad r = 1{,}5 - 1 \qquad r = 0{,}5$$

b) SQUARE

$$r = \frac{n}{2} - 1 \qquad r = \frac{4}{2} - 1 \qquad r = 2 - 1 \qquad r = 1$$

c) PENTAGON

$$r = \frac{n}{2} - 1 \qquad r = \frac{5}{2} - 1 \qquad r = 2{,}5 - 1 \qquad r = 1{,}5$$

TABLE 1

RATIO BETWEEN THE INNER ANGLE AND THE CENTRAL ANGLE OF A REGULAR POLYGON

POLYGON	RATIO
TRIANGLE	0,5
SQUARE	1
PENTAGON	1,5
HEXAGON	2
HEPTAGON	2,5
OCTAGON	3
ENNEAGON	3,5
DECAGON	4

ACCORDING TO TABLE 1 ABOVE, WE REALIZE THAT THE VALUE OF RATIO "r" INCREASES BY 0,5 WHEN THE NUMBER OF SIDES OF THE POLYGON INCREASES BY 1. THIS CONCLUSION PERMITS THE FOLLOWING EXPRESSION:

$$r = n \cdot 0,5 - 1 \qquad \text{or} \qquad r = \frac{n}{2} - 1$$

AS WE CAN SEE, THIS IS THE SAME EQUATION Nº (6).

SO, WE CAN STATE:

"THE RATIO BETWEEN THE INNER ANGLE AND THE CENTRAL ANGLE OF A REGULAR POLYGON IS EQUAL TO THE HALF OF THE NUMBER OF SIDES, MINUS 1 (ONE)"

2.2 - EXAMPLES

APPLICATION OF THE THEOREM

a) EQUILATERAL TRIANGLE

DATA

$\hat{IA} = 60°$

$\hat{CA} = 120°$

$\dfrac{\hat{IA}}{\hat{CA}} = 0,5$

SOLUTION

$n = 2 \cdot r + 2$

$n = 2 \cdot 0,5 + 2$

$n = 1 + 2$

$n = 3$

b) SQUARE

DATA

$\hat{IA} = 90°$

$\hat{CA} = 90°$

$\dfrac{\hat{IA}}{\hat{CA}} = 1$

SOLUTION

$n = 2 \cdot r + 2$

$n = 2 \cdot 1 + 2$

$n = 2 + 2$

$n = 4$

c) PENTAGON

DATA

$\hat{IA} = 108°$

$\hat{CA} = 72°$

$\dfrac{\hat{IA}}{\hat{CA}} = 1,5$

SOLUTION

$n = 2 \cdot r + 2$

$n = 2 \cdot 1,5 + 2$

$n = 3 + 2$

$n = 5$

d) HEXAGON

DATA	SOLUTION
$\hat{IA} = 120^\circ$	$n = 2 \cdot r + 2$
$\hat{CA} = 60^\circ$	$n = 2 \cdot 2 + 2$
$\frac{\hat{IA}}{\hat{CA}} = 2$	$n = 4 + 2$
	$n = 6$

3 - CONCLUSION

ALTHOUGH THERE EXISTS A FORMULA TO DETERMINATE THE NUMBER OF SIDES OF A REGULAR POLYGON, BASED IN THE CENTRAL ANGLE, THE THEOREM REFERED IN THIS CASE SIMPLIFIED THE CALCULATIONS OF MOST COMPLEXED PROBLEMS IN ELEMENTARY GEOMETRY.

BASED ON THIS THEOREM, IT WAS POSSIBLE TO DEDUCT A FORMULA TO CALCULATE THE RATIO BETWEEN THE INNER ANGLE AND THE CENTRAL ANGLE OF A REGULAR POLYGON.

IT'S EXPECTED THAT, IN A NEAR FUTURE, THE PRESENTED THEOREM, NOT ONLY HELPS TO CALCULATE ELEMENTARY GEOMETRY, BUT ALSO IN VARIOUS TECHNICAL AND SCIENTIFIC FIELDS

4 - BIBLIOGRAPHICAL REFERENCES

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